

Comment on the paper “Does Zeeman’s Fine Topology Exist?” at arXiv:1003.3703v1 [math-ph]

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A constructive and straightforward proof of the existence of the Zeeman topology is provided, contradicting a fallacious claim contained in the paper “Does Zeeman’s Fine Topology Exist?” available at arXiv:1003.3703v1.

Preamble

In the past few years two papers appeared on the arXiv claiming to prove the non-existence of the Zeeman topology. The oldest one [3] has been withdrawn and replaced by a new one [4] which shares techniques and conclusions with the oldest. A review of these papers is beyond the scope of this short communication.

Suffice to say that the existence of Zeeman’s topology is made obvious by a reading of the original and beautiful paper [5]. For the sake of clarity a constructive proof of its existence is spelt out below, essentially rephrasing an argument of Zeeman.

Constructive proof of the existence of Zeeman’s topology

The Zeeman topology on Minkowski space M is defined to be the finest topology on M inducing¹ the Euclidean topology on each timelike and spacelike affine subspace (from now on called axes). To prove its existence, let us consider the collection C of all subsets of M that meet each axis in an open² set. This collection satisfies the axioms for a topology (straightforward exercise, see Lemma III.3 in [1]) and induces the Euclidean topology on each axis by construction. Given a topology T on M inducing the Euclidean topology on each axis, any element of T meets each axis in an open set, thereby T is coarser than C and the proof is complete.

Side note about Larson’s theorem

In [4], Larson’s theorem about maximum and minimum topological spaces is quoted. Let us review this interesting theorem (see [2]). Fix a topological property P . A topology T on some set X is called *maximum P* [*minimum P*] when any topology on X with property P is coarser [finer] than T . Larson’s theorem states that a topology on X is maximum P or minimum P for some topological property P if and only if each bijection of X onto itself is a homeomorphism. Contrary to claims in [4], this theorem does not apply to Zeeman’s topology since the property of inducing the Euclidean topology on each axis is not a topological property (this is obvious since its very formulation requires the concept of affine subspace, which is not topologically invariant).

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- [1] G. Dossena, *Some results on the Zeeman topology*, J. Math. Phys. 48, 113507 (2007)
 - [2] R. E. Larson, *Minimum and maximum topological spaces*, Bull. Pol. Acad. Sci., 18:707–710 (1970)
 - [3] N. B. Sáinz, *Inexistence of Zeeman’s fine topology* (arXiv, 2008); available at arXiv:0803.3352v1 [math-ph]
 - [4] N. B. Sáinz, *Does Zeeman’s Fine Topology Exist?* (arXiv, 2010); available at arXiv:1003.3703v1 [math-ph]
 - [5] E. C. Zeeman, *The Topology of Minkowski Spacetime*, Topology 6, 161–170 (1967)

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¹ Given a topological space (X, T) and a subset $Y \subset X$ with a topology T_Y , we say (X, T) induces T_Y on Y if (Y, T_Y) is a subspace of (X, T) . In other words, each open set in (Y, T_Y) is the intersection of Y with an open set in (X, T) .

² Open w.r.t. the Euclidean topology of the axis, of course.